**Some Counterparty Risk Examples**

Suppose that a stock follows a lognormal process

and we have a position in a call option on S, strike K, risk-free rate r, dividend rate y, expiration T and volatility Suppose this option is over the counter with a counterparty that has a constant hazard rate and recovery rate R. Then the **exposure** at future time is where

In this case we don’t need the + in the superscript since always.

We can compute the **Expected Exposure (EE)** at time t by computing

To do this we can note that this essentially the same has having a call payoff at time T, with the real-world drift of on the interval and the risk-neutral drift of in the interval but with discounting only on This results in a modified Black-Scholes European call value given by

We can now compute the **Potential Future Exposure (PFE)** at each time t. Consider a percentile Our first step is to find such that

using the real-world measure. This translates to

At time t, the value of the call option position for is now

If we want the **Maximum PFE,** we take the maximum of this expression over all

The **CVA** for this sort of position has a particularly simple form. In the risk-neutral measure, it is always true for a call option that

Thus

Now suppose instead of a call option, you have a *forward contract* expiring at T, which is just a call minus a put at the same strike. This will give somewhat different results. We will note that these calculations introduce “optionality” even though the underlying position has none. Start with

The **Expected Exposure** now becomes

We now let this becomes

Then the **PFE** will just be

The **CVA** will come out different also. At time t you have optionality out to t, not T, so that you get

Except that now looks just like EE(t) but with r in place of that is